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**PROBLEMS IN NONLINEAR ACOUSTICS:**

**PULSED FINITE AMPLITUDE SOUND BEAMS,  
NONLINEAR ACOUSTIC WAVE PROPAGATION IN A LIQUID LAYER,  
NONLINEAR EFFECTS IN ASYMMETRIC CYLINDRICAL SOUND BEAMS,  
EFFECTS OF ABSORPTION ON THE INTERACTION OF SOUND BEAMS,  
AND PARAMETRIC RECEIVING ARRAYS**

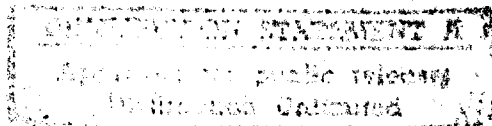
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19 ABSTRACT (Continue on reverse if necessary and identify by block number)  Five projects are discussed in this annual summary report, all of which involve basic theoretical research in nonlinear acoustics. (1) <u>Pulsed Finite Amplitude Sound Beams</u> are studied with a recently developed time domain computer algorithm that solves the KZK nonlinear parabolic wave equation. (2) <u>Nonlinear Acoustic Wave Propagation in a Liquid Layer</u> is a study of harmonic generation and acoustic soliton formation in a liquid between a rigid and a free surface. (3) <u>Nonlinear Effects in Asymmetric Cylindrical Sound Beams</u> is a study of source asymmetries and scattering of sound by sound at high intensity. (4) <u>Effects of Absorption on the Interaction of Sound Beams</u> is a completed study of the role of absorption in second harmonic generation and scattering of sound by sound. (5) <u>Parametric Receiving Arrays</u> is a completed study of parametric reception in a reverberant environment.					
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## INTRODUCTION

This annual summary report describes research performed from 1 August 1989 through 30 September 1990 (14 months) with support from ONR under grant N00014-89-J-1003. The following projects are discussed in this report:

- I. Pulsed Finite Amplitude Sound Beams
- II. Nonlinear Acoustic Wave Propagation in a Liquid Layer
- III. Nonlinear Effects in Asymmetric Cylindrical Sound Beams
- IV. Effects of Absorption on the Interaction of Sound Beams
- V. Parametric Receiving Arrays

Contributions to these projects were made by the following individuals:

### Senior Personnel

- M. F. Hamilton, principal investigator (projects I-V)
- J. Naze Tjøtta, visiting scientist (projects IV and V)
- S. Tjøtta, visiting scientist (project IV)
- E. A. Zabolotskaya, visiting scientist (project II)

### Graduate Students

- C. M. Darvennes, Ph.D. student in Mechanical Engineering during August-December 1989, postdoctoral research fellow during January-August 1990 (projects IV and V)
- E. E. Kim, M.S. student in Mechanical Engineering (project III)
- Y.-S. Lee, Ph.D. student in Mechanical Engineering (project I)

Professor Naze Tjøtta spent the entire 14 months covered by this report, and Professor Tjøtta spent four months, at Applied Research Laboratories at the University of Texas at Austin (ARL:UT) while on leave from the University of Bergen, Norway. The Tjøttas received no support from the ONR grant covered by this report. Their support on the above projects was provided by ARL:UT, the Texas Advanced Research Program (TARP), VISTA/STATOIL of Norway, and the Odd Hassel Fellowship of the Norwegian Research Council for Science and Humanities (only JNT). Professor Zabolotskaya spent five months in the Mechanical Engineering Department while on leave from the Institute of General Physics in Moscow, USSR. Although she received her entire support from the David and Lucile Packard

Fellowship for Science and Engineering, her research was performed in collaboration with Hamilton, whose work was supported in part by ONR. Kim received partial support from TARP (2 months), while Lee received his primary support from TARP (11 months). Hamilton also received partial financial support from TARP.

The following manuscripts and abstracts, which describe work supported at least in part by ONR, have been published (or submitted for publication) since 1 August 1989.

### Refereed Publications

- C. M. Darvennes and M. F. Hamilton, "Scattering of sound by sound from two Gaussian beams," *J. Acoust. Soc. Am.* **87**, pp. 1955-1964 (1990).
- C. M. Darvennes, M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Effects of absorption on the nonlinear interaction of sound beams," submitted in April 1990 for publication in *J. Acoust. Soc. Am.*
- M. F. Hamilton and D. T. Blackstock, "On the linearity of the momentum equation for progressive plane waves of finite amplitude," submitted in June 1990 for publication in *J. Acoust. Soc. Am.*
- M. F. Hamilton and E. A. Zabolotskaya, "Nonlinear acoustic wave propagation in a liquid layer between a rigid and a free surface," submitted in June 1990 for publication in *J. Acoust. Soc. Am.*

### Publications in Conference Proceedings

- C. M. Darvennes, M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Scattering of sound by sound: Effects of absorption," in *Proceedings of the 13th International Congress on Acoustics*, Belgrade, Yugoslavia, August 1989, edited by P. Pravica and G. Drakulic (Sava Centar, Belgrade, 1989), Vol. 1, pp. 283-286.
- M. F. Hamilton, "Finite amplitude sound in waveguides," Compilation of Abstracts, XIII Scandinavian Cooperation Meeting in Acoustics/Hydrodynamics, edited by H. Hobæk (Scientific/Technical Report No. 227, Department of Physics, University of Bergen, Norway, 1990), pp. 4-9.
- C. M. Darvennes, M. F. Hamilton, and J. Naze Tjøtta, "Parametric reception near a reflecting surface," *Frontiers of Nonlinear Acoustics: 12th ISNA*, edited by M. F. Hamilton and D. T. Blackstock (Elsevier, London, 1990), pp. 239-244.

- M. F. Hamilton and E. A. Zabolotskaya, "Nonlinear wave propagation in a fluid layer," *Frontiers of Nonlinear Acoustics: 12th ISNA*, edited by M. F. Hamilton and D. T. Blackstock (Elsevier, London, 1990), pp. 321-326.

### Oral Presentation Abstracts

- C. M. Darvennes, M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Effects of focusing on the scattering of sound by sound," *J. Acoust. Soc. Am.* **86**, S106 (1989).
- C. M. Darvennes, M. F. Hamilton, and J. Naze Tjøtta, "Parametric reflection near a reflecting surface," *J. Acoust. Soc. Am.* **87**, S21 (1990).
- M. F. Hamilton and E. A. Zabolotskaya, "Nonlinear wave propagation in a fluid layer," *J. Acoust. Soc. Am.* **87**, S21 (1990).

## I. PULSED FINITE AMPLITUDE SOUND BEAMS

A time domain computer algorithm for solving the KZK (Khokhlov-Zabolotskaya-Kuznetsov) nonlinear parabolic wave equation has been developed for pulsed, axisymmetric, finite amplitude sound beams in thermoviscous fluids. This work has been performed by Lee.

### A. Background

The KZK equation accounts consistently for the combined effects of nonlinearity, diffraction, and thermoviscous absorption on sound beams radiated by directive sources. Computer algorithms that are currently used to solve the fully nonlinear KZK equation employ the Fourier series method that was developed originally by Aanonsen et al.,<sup>1,2</sup> and all computations are thus performed in the frequency domain. The frequency domain algorithms have been used with great success for monofrequency and bifrequency sources. However, as either the source amplitude or the number of source frequencies is increased, more harmonics must be retained in the Fourier series, and the computation time increases accordingly. Baker and Humphrey,<sup>3</sup> and Neighbors and Bjørnø,<sup>4</sup> have recently used the Aanonsen algorithm to model the nonlinear propagation of pulsed sound beams. To reduce the number of harmonics required to adequately describe a single pulse, they employ an infinite sequence of pulses spaced sufficiently far apart in time that adjacent pulses do not interact with each other. Again, however, increasing the complexity of the pulse shape generally requires more harmonics and therefore greater computation time. By performing all calculations in the time domain, our algorithm avoids the limitations imposed by using a truncated Fourier series in frequency domain calculations. Further background and motivation for this project are covered in the First Annual Summary Report<sup>5</sup> for ONR Grant N00014-89-J-1003.

## B. Results

The numerical solutions of the KZK equation are obtained from a computer algorithm, developed by Lee, that performs all calculations in the time domain. A transformation is applied to the KZK equation in order to facilitate calculations in the farfield,<sup>6</sup> and the resulting parabolic equation is integrated once analytically with respect to time to yield

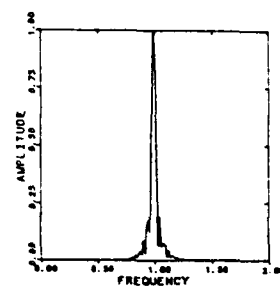
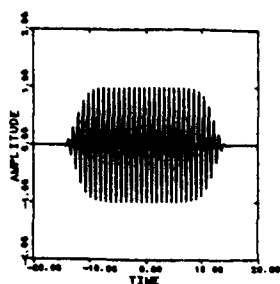
$$\frac{\partial P}{\partial \sigma} = \frac{1}{4(1+\sigma)^2} \int (\nabla_{\perp}^2 P) d\tau + A \frac{\partial^2 P}{\partial \tau^2} + \frac{BP}{(1+\sigma)} \frac{\partial P}{\partial \tau}, \quad (1)$$

where  $P$ ,  $\sigma$ , and  $\tau$  are dimensionless quantities for the pressure, range from the source, and retarded time, respectively. The Laplacian  $\nabla_{\perp}^2$  in the diffraction term operates in the plane that is transverse to the axis of the beam,  $A$  is an absorption coefficient, and  $B$  is a nonlinearity coefficient. Finite difference methods are used to integrate the diffraction and absorption terms over each incremental step from  $\sigma$  to  $\sigma + \Delta\sigma$ . Nonlinearity is included by changing the phase  $\tau_i$  of the  $i$ th point on the waveform to  $\tau_i - BP \ln[1 + \Delta\sigma/(1 + \sigma)]$ , which yields an exact solution of the above equation when the diffraction and absorption terms are ignored ( $\nabla_{\perp}^2 P = 0$  and  $A = 0$ ). Weak shock theory may also be included in the algorithm,<sup>7</sup> and focusing may be taken into account through the source condition.<sup>8</sup>

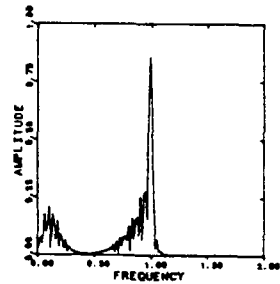
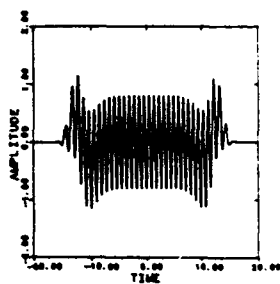
Numerical results for the self-demodulation of a tone burst radiated by a uniform piston are shown in Fig. 1. In the left column are shown the time waveforms along the axis of the beam, and in the right column are shown the frequency spectra that are obtained by taking the Fourier transform of the corresponding waveforms. The dimensionless range  $\sigma$  is the ratio of the range along the axis of the sound beam to the Rayleigh distance of the tone. The source strength used in the example would cause the tone to form a shock at  $\sigma = 1.0$  in the absence of both diffraction (i.e., for a plane wave) and absorption. The absorption length  $\alpha^{-1}$ , where  $\alpha$  is the attenuation coefficient for the tone, is  $\sigma = 0.5$  in terms of the dimensionless range variable. As the pulse propagates into the farfield, energy is shifted downward toward a frequency that characterizes the distorted envelope of the pulse, in general agreement with the pioneering asymptotic predictions by Berkta. Future work will involve comparisons of the results for self-demodulation with asymptotic predictions not only by Berkta, but more recently by Frøysa et al.<sup>10,11</sup> However, the numerical results are not subject to several restrictions that apply to the theoretical analyses, e.g., farfield and weak nonlinearity. Additional investigations will include strong nonlinearity relative to absorption, and frequency modulation of the tone.

# TIME WAVEFORMS

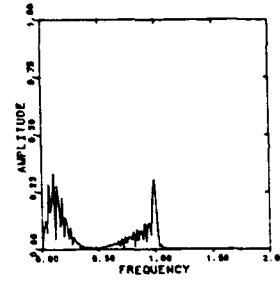
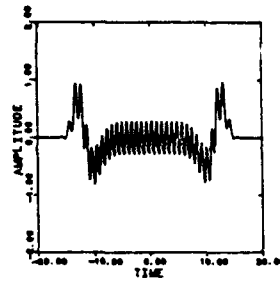
# FREQUENCY SPECTRA



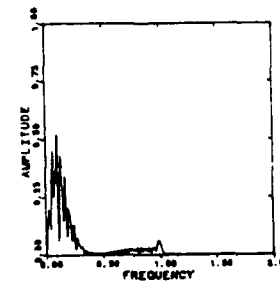
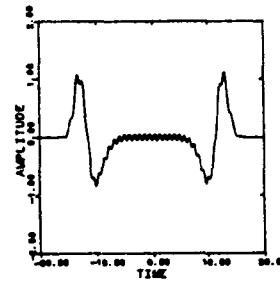
$\sigma = 0.$   
(1 : 1)



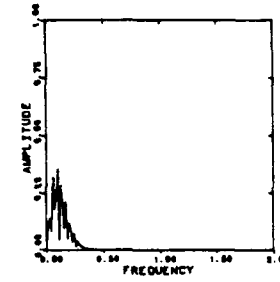
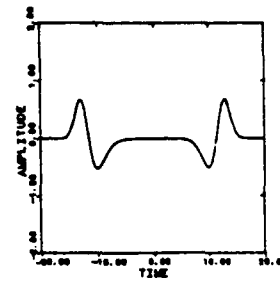
$\sigma = 4.1$   
(1 :  $10^{-4}$ )



$\sigma = 5.0$   
(1 :  $5 \cdot 10^{-5}$ )



$\sigma = 6.2$   
(1 :  $2.5 \cdot 10^{-5}$ )



$\sigma = 8.5$   
(1 :  $2.5 \cdot 10^{-5}$ )

Figure 1: Numerical simulation of self-demodulation of a tone burst.



## II. NONLINEAR ACOUSTIC WAVE PROPAGATION IN A LIQUID LAYER

This theoretical investigation was performed in collaboration with Zabolotskaya.

### A. Background

In contrast to the propagation of finite amplitude sound in unbounded homogeneous media, the nonlinear propagation of sound in waveguides and layered media is strongly affected by dispersion. The dispersion law is determined by the boundary conditions at the interfaces and the inhomogeneity of the medium, and it can substantially alter the processes of harmonic generation and waveform distortion. Natural acoustic waveguides include<sup>12,13</sup> shallow water channels, isovelocity channels in deep ocean, layers in the earth, and atmospheric waveguides.

In the present investigation, the nonlinear propagation of two dimensional sound waves in a lossless and homogeneous liquid layer are considered. The layer is bounded below by a rigid surface and above by a free surface, and it thus approximates the conditions in a shallow water channel. The free surface prevents the mode coupling that can lead to shock formation in rectangular ducts with rigid walls. Investigations of wave distortion and shock formation in ducts with rigid walls are the subjects of many papers that have been published during the past two decades (see, e.g., the list of articles in Ref. 14).

Previous papers<sup>15-19</sup> on finite amplitude sound in shallow water channels are concerned primarily with parametric (difference frequency) amplification and radiation due to the interaction of two tones. In Refs. 15 and 16 the primary waves are assumed to propagate in single modes, and a main objective of the investigations is to identify resonance conditions for efficient parametric amplification. In Refs. 17-19 it is assumed that the primary waves form highly collimated beams which are absorbed before diffraction effects become significant.

### B. Results

The following results have been excerpted from Ref. 24.

First, a solution is derived for second harmonic generation due to the propagation of a tone in a single mode. The linear solution which satisfies the boundary conditions for a free surface at  $x = 0$  and a rigid surface at  $x = h$  may be written in the form

$$p_1 = p_0 \sin(\gamma_m x) e^{j(\omega t - k_z z)}, \quad (2)$$

where  $p_0$  is the characteristic source pressure,  $\gamma_m = (2m - 1)\pi/2h$  defines the mode shape,  $k_z = (\omega/c_0)\sqrt{1 - (\omega_m/\omega)^2}$  is the propagation wavenumber,  $c_0$  is the small signal sound speed, and  $\omega_m = \gamma_m c_0$  is the cutoff frequency for the  $m$ th mode

( $m = 1, 2, \dots$ ). We consider separately the particular and homogeneous solutions for the second harmonic component, and we write

$$p_2 = (p_2)_p + (p_2)_h. \quad (3)$$

The particular solution, with  $p_1$  given by Eq. (2), is

$$(p_2)_p = \frac{p_0^2 \omega^2}{\rho_0 c_0^2 \omega_m^2} \left[ \left( \frac{c_0^4}{c_z^4} + \frac{B}{2A} \right) \sin^2(\gamma_m x) - \frac{\gamma_m}{2} \left( 1 + \frac{B}{2A} \right) (x - h) \sin(2\gamma_m x) \right] e^{j2(\omega t - k_z z)}, \quad (4)$$

where  $\rho_0$  is the static density,  $B/A$  is a ratio associated with the quadratic non-linearity in the isentropic equation of state that relates the pressure and density, and  $c_z = \omega/k_z$  is the phase speed in the  $z$  direction. The mode shapes for the forced second harmonic wave  $(p_2)_p$  are shown in Fig. 2 (solid lines). Also shown are the corresponding mode shapes for the fundamental wave  $p_1$  (dashed lines), which propagates in water ( $B/A = 5$ ), in the  $m$ th mode, at twice the cutoff frequency ( $\omega = 2\omega_m$ ). For  $m > 1$ , second harmonic generation is seen to be most efficient near the free surface.

The homogeneous solution is of the general form

$$(p_2)_h = \sum_{n=1}^{\infty} b_n \sin(\gamma_n x) e^{j2(\omega t - \kappa_n z)}, \quad (5)$$

where  $\kappa_n = (\omega/c_0) \sqrt{1 - (\omega_n/2\omega)^2}$  is the propagation wavenumber for the  $n$ th mode. The summation includes both propagating ( $\omega_n < 2\omega$ ) and evanescent ( $\omega_n > 2\omega$ ) waves. The propagating waves have different phase speeds ( $\omega/\kappa_n$ , where  $n = 1, 2, \dots$ ), not one of which is equal to the phase speed of the forced wave ( $c_z = \omega/k_z$ ). The constants  $b_n$  are determined by the source condition, which is assumed to be  $p_2 = 0$  at  $z = 0$ .

Shown in Fig. 3 are propagation curves and mode shapes for  $|p_2|$  obtained from Eqs. (3)–(5) with  $m = 2$ ,  $\omega = 2\omega_m$ , and  $B/A = 5$ . Range is measured in terms of the

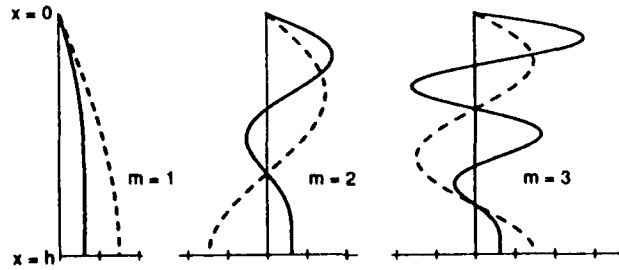


Figure 2: Mode shapes for  $p_1$  (dashed lines) and  $(p_2)_p$  (solid lines).

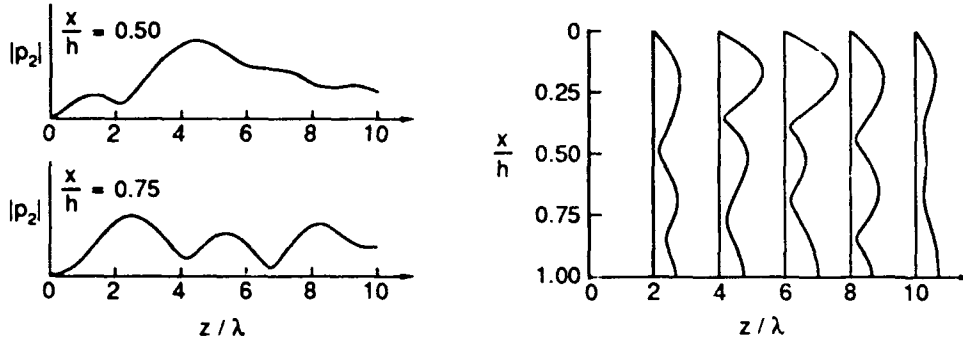


Figure 3: Propagation curves (left) and mode shapes (right) for  $|p_2|$ .

wavelength  $\lambda = 2\pi c_0/\omega$  at the fundamental frequency. The homogeneous solution  $(p_2)_h$  accounts for unforced (i.e., free) second harmonic waves that propagate in the six lowest modes ( $1 \leq n \leq 6$ ) and beat with the forced wave  $(p_2)_p$ . The evanescent waves described by the homogeneous solution ( $n \geq 7$ ) are required to satisfy the source condition  $p_2 = 0$  at  $z = 0$ . Dispersion causes the mode shapes to vary with range.

We now consider the asymptotic properties of a slowly varying envelope that modulates a high frequency tone. It is assumed that the fundamental wave may be described by

$$p_1 = a(\epsilon\tau, \epsilon^2 z) \sin(\gamma_m x) e^{j(\omega t - k_z z)}, \quad (6)$$

where  $a(\epsilon\tau, \epsilon^2 z)$  is a slowly varying envelope with magnitude of order  $\epsilon$ , and  $\tau = t - z/c_g$  is a retarded time based on the group velocity  $c_g = d\omega/dk_z$ . The envelope varies more rapidly with time than with range, as indicated by the explicit dependence on the combinations  $\epsilon\tau$  and  $\epsilon^2 z$ . The slow scales  $\tau_1 = \epsilon\tau$  and  $z_1 = \epsilon^2 z$  may be introduced to yield, at order  $\epsilon^3$ , the following nonlinear Schrödinger equation for the pulse envelope function:

$$j \frac{\partial a}{\partial z} + \frac{k_z''}{2} \frac{\partial^2 a}{\partial \tau^2} + \Lambda |a|^2 a = 0, \quad (7)$$

where  $k_z'' = d^2 k_z / d\omega^2$ , and  $\Lambda$  is determined by the nonlinear interaction of the primary and second harmonic waves. Solutions of Eq. (7) are well known<sup>25,26</sup> and are classified differently depending on the sign of  $\Lambda/k_z''$ . With  $\Lambda/k_z'' < 0$ , the modulation is said to be stable, and the amplitude of the pulse decays according to  $|a| \propto 1/\sqrt{z}$ . With  $\Lambda/k_z'' > 0$ , the modulation is unstable, and the pulse may evolve into a series of envelope solitons. The solution for the amplitude of a single soliton is  $|a| = P_0 \text{sech}(\tau/T)$ , where  $P_0$  is a characteristic amplitude, and  $T = P_0^{-1} \sqrt{k_z''/\Lambda}$  is the corresponding period. For all typical liquids we found  $\Lambda/k_z'' < 0$ , and pulse propagation is therefore stable. For the special case where  $|a|$  is finite at  $\tau = \pm\infty$ , kink solitons described by  $|a| = P_0 \tanh(|\tau|/T)$  can exist with  $\Lambda/k_z'' < 0$ .

### III. NONLINEAR EFFECTS IN ASYMMETRIC CYLINDRICAL SOUND BEAMS

This project was completed by Kim in August 1990, at which time he completed his M.S. thesis<sup>27</sup> on the results from this work. The following background and results have been excerpted from his M.S. thesis.

#### A. Background

Much of the work in the field of acoustics has concentrated on sound sources that generate axisymmetric sound fields. However, problems that involve nonaxisymmetric sound fields include propagation and reflection of sound in shallow water, caustic formation in inhomogeneous media, scattering of sound by sound, and rectangular sources used in medicine and in underwater applications.

A goal of this investigation was to develop a relatively simple numerical model of nonlinear effects in asymmetric sound beams. Until recently, practical constraints on computation time have limited numerical analyses of diffracting finite amplitude sound beams to axisymmetric source conditions. Specifically, an axisymmetric sound beam may be described with only two spatial coordinates, i.e., the cylindrical coordinates  $(r, z)$ , where  $z$  measures range along the axis of the beam, and  $r$  measures distance from the  $z$  axis. In contrast, a nonaxisymmetric beam must be described with three spatial coordinates. The computer program that forms the basis of the numerical analysis is an extension of the Aanonsen frequency domain algorithm (see Section I) due to Berntsen and Vefring.<sup>28</sup> The program solves the KZK equation [Eq. (1) in Section I] for axisymmetric sound beams, and in principle, the algorithm is easily modified for applications to nonaxisymmetric sound beams. However, the computation time required for nonaxisymmetric beams would be roughly the square of that required for axisymmetric beams. Work is currently underway at the University of Bergen, Norway, to reduce the computation time required to use the program for general nonaxisymmetric fields. Several examples of nonaxisymmetric sound fields include parametric transmitting<sup>29,30</sup> and receiving<sup>31,32</sup> arrays, oblique transmission of a beam through an interface,<sup>33</sup> scattering of sound by sound,<sup>34,35</sup> and wave propagation in inhomogeneous fluids.<sup>36</sup>

The numerical computation time in analyses of nonaxisymmetric sound beams can be reduced considerably by reducing the dimension of the computation field. To this end, a two-dimensional sound field is used to study nonaxisymmetric nonlinear wave propagation. In spite of the physical shortcomings of a two-dimensional sound field in relation to a three-dimensional sound field (i.e., cylindrical as opposed to spherical spreading), the two-dimensional field is nevertheless of practical interest. For example, cylindrical spreading is often an appropriate model for waveguides, ocean acoustics, and the nearfields of cylindrical sources. Moreover, recent studies of caustic formation by Marston,<sup>37,38</sup> and of underwater propagation of pulses by

McDonald and Kuperman,<sup>39</sup> are performed for two-dimensional sound fields. In cases where cylindrical spreading is an inappropriate approximation of a given finite amplitude sound field, the analysis in two-dimensions should still provide physical insight into phenomena which occur in the corresponding spherically spreading field.

## B. Results

Closed form analytical solutions were derived for linear radiation from an amplitude shaded piston (two dimensional), and for second harmonic generation in a Gaussian beam. The analytical solutions, which were derived from the KZK equation, were used to verify the accuracy of the numerical solutions, and to gain insight into the asymptotic properties of cylindrically spreading sound beams. Two problems were studied in detail with the computer program, high intensity radiation from amplitude shaded piston sources, and scattering of sound by sound from two Gaussian beams.

Graphical results from the investigation of amplitude shaded pistons were presented in the First Annual Summary Report.<sup>5</sup> The results show that nonlinear effects are altered by asymmetric source conditions. Namely, the definition of the sidelobe structure in the higher harmonic beam patterns, including the nonlinear nearfield effects known as fingers,<sup>6</sup> are very sensitive to source asymmetry. We also observed, in the results from both nonlinear and linear theories, that the degree of asymmetry in the beam decreases with range from the source.

The fundamental aspects of scattering of sound by sound were found to be very similar in both spherically spreading and cylindrically spreading primary beams. This conclusion was reached by comparing quasilinear numerical results for scattering of cylindrical Gaussian beams with quasilinear analytical results for scattering of spherical Gaussian beams.<sup>35</sup> Specifically, the angle at which scattered sound is most prominent is the same in the two cases, and the dependence of the relative amplitude of the scattered sound on various source parameters (e.g., beam intersection angle, frequency ratio, etc.) also follows the same general trends in each case. Fully nonlinear scattering of sound by sound was also investigated numerically. We observed that noncollinear interaction of sound beams may produce highly oscillatory beam patterns due to interference between harmonics that propagate in different directions. Higher order scattering that is not predicted by quasilinear theory was also observed. In special cases, however, quasilinear theory was found to be instrumental in predicting the direction of the higher order scattering.

## IV. EFFECTS OF ABSORPTION ON THE INTERACTION OF SOUND BEAMS

This work was performed by Darvennes, Naze Tjøtta, and Tjøtta as an extension of earlier research on scattering of sound by sound, as discussed in the First Annual Summary Report.<sup>5</sup> This project is now completed.

### A. Background

In the previous annual summary report, the effects of absorption on the nonlinear interaction of two Gaussian beams were discussed. The results of that work were reported in Ref. 40. During the past year, the numerical results were extended to include second harmonic generation in a sound beam radiated by a circular piston.

### B. Results

The following summary is excerpted from a paper<sup>41</sup> that is currently in press.

Asymptotic formulas were derived for the sum and difference frequency sound that is generated by the interaction of two harmonic sound beams in a dissipative fluid. The results are valid for arbitrary source configurations, within the limits of the parabolic approximation. At range  $z$ , different asymptotic results apply to the cases of moderate absorption [ $z = O(|L_{\pm}|) \gg R_{\pm}$ ] and strong absorption ( $z \gg R_{\pm} \gg |L_{\pm}|$ ), where  $L_{\pm} = (\alpha_1 + \alpha_2 - \alpha_{\pm})^{-1}$  is an absorption length,  $R_{\pm} = 2k_1 k_2 a^2 / k_{\pm}$  is a diffraction length,  $k$  and  $\alpha$  are the wavenumbers and absorption coefficients, respectively, at the primary frequencies  $\omega_1$  and  $\omega_2$  and the sum and difference frequencies  $\omega_{\pm} = \omega_1 \pm \omega_2$ , and  $a$  is the characteristic radius of the source. It was found that the relative amplitudes of the nonlinearly generated pumped and scattered waves (see Refs. 34 and 35 for precise definitions of the pumped and scattered waves) depend not only on the nature and geometry of the sources (e.g.,  $ka$  of each source, on-source pressure distributions, source separation, and beam interaction angle), but also on the dissipative properties of the medium through the sign of  $L_{\pm}$ . In the farfield, the scattered sound dominates the pumped sound at combination frequencies for which  $L_{\pm} > 0$ , whereas the reverse is true at combination frequencies for which  $L_{\pm} < 0$ . Numerical results have been obtained for the case of two Gaussian beams that cross at an angle, and for the second harmonic component generated by a circular uniform piston source. Various frequency dependencies were considered for the absorption law: quadratic, linear, and square root.

## V. PARAMETRIC RECEIVING ARRAYS

This project, which is now completed, was performed in collaboration with Darvennes and Naze Tjøtta.

### A. Background

The motivation for this research is the potential use of a parametric receiving array for measuring freefield source directivities in reverberant environments. Background on this project may be found in Ref. 31.

### B. Results

New results during the past year have included the effects of absorption on parametric reception. The following summary is excerpted from a paper<sup>42</sup> that was presented at the 12th International Symposium on Nonlinear Acoustics in August 1990.

Beam patterns are obtained with a parametric receiving array (parray) by rotating a high frequency ( $\omega_1$ ) pump beam through the entire low frequency ( $\omega_2$ ) wave field, with the hydrophone always located on the axis of the pump beam. For our application, the pump is located at the center of a low frequency source, and the sum and difference frequency pressure  $p_{\pm}(\theta, z)$  is recorded at the hydrophone. When there is no absorption and no reflector, the pressure  $p_{\pm}(\theta, z)$  exhibits an angular dependence that converges as  $z \rightarrow \infty$  to the product of the freefield directivity functions of the primary beams,  $D_1(\theta)D_2(\theta)$ . Given the known pump directivity  $D_1(\theta)$ , an estimate of the unknown source directivity  $D_2(\theta)$  may thus be obtained from the normalized pressure  $P_{\pm}(\theta, z) = p_{\pm}(\theta, z)/D_1(\theta)$ . In the absence of absorption, however,  $P_{\pm}(\theta, z)$  is a good approximation of the farfield directivity function  $D_2(\theta)$  only when  $\ln(z/z_j) \gg 1$  (where  $j = 1, 2$ , and  $z_j$  is the Rayleigh distance of the  $j$ th primary beam, i.e., of either the low frequency source or the pump). In the presence of a reflector and/or of dissipation, the relation between  $P_{\pm}(\theta, z)$  and  $D_2(\theta)$  may be more complex.

As discussed in the previous annual summary report,<sup>5</sup> the parray is capable of discriminating against multipath interference due to reflections from an adjacent surface. However, the efficiency of the discrimination depends on the distance between the pump and the hydrophone. As the distance is increased, the multipath components become increasingly significant. When the hydrophone is close to the source, the interference is less significant, but  $P_{\pm}(\theta, z)$  slightly overestimates the beamwidth of  $D_2(\theta)$ . The overestimation of beamwidth due to finite hydrophone ranges is discussed in detail in Ref. 6.

In Fig. 4 we show the effect of thermoviscous absorption on the normalized sum and difference frequency pressures  $P_{\pm}(\theta, z)$ . The absorption increases from left to right in the figures, with Fig. 4(a) showing the case of no absorption. The

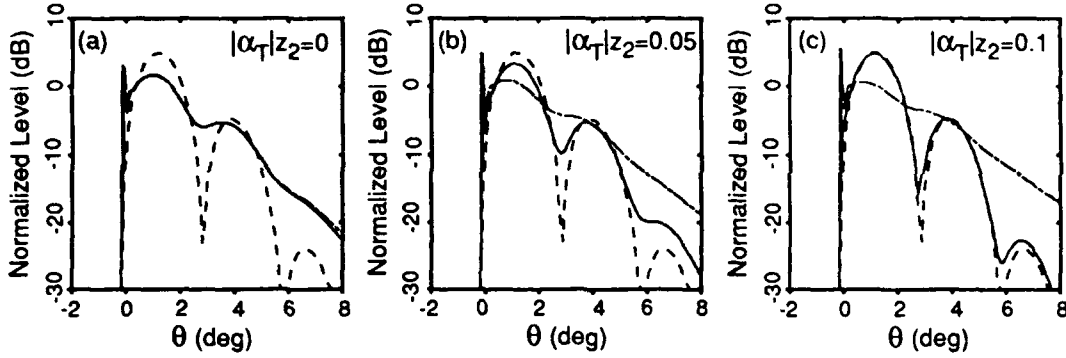


Figure 4: Effect of absorption on parametric reception near a reflecting surface.

sum frequency beam pattern  $P_+(\theta, z)$  is the solid line, the difference frequency beam pattern  $P_-(\theta, z)$  is the dot-dash line, and the low frequency beam pattern (that which is to be measured with the pararray) is the dashed line. The range is  $z = 50z_2$  and the source depth is given by  $h = 2\varepsilon_2$ , where  $\varepsilon_2$  is the source radius, and  $k_1/k_2 = 100$ ,  $\varepsilon_2/\varepsilon_1 = 10$ ,  $k_2\varepsilon_2 = 30$ . As the absorption parameter  $|\alpha_T|z_2$  is increased from 0 to 0.1, the sum frequency directivity  $P_+(\theta, z)$  tends toward the low frequency beam pattern. In contrast, for large values of absorption,  $P_-(\theta, z)$  tends toward a different directivity function which depends primarily on the ratio  $k_-/\alpha_T$ . The sum frequency signal, rather than the difference frequency signal, therefore provides more information about the low frequency source distribution when absorption is significant. This conclusion also follows from the more general analysis of absorption in Section IV of this report.



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